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Optimal scheduling of batch industrial facilities

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An increasing interest in batch processing has been evident in recent years. This renewed interest is explained by the inherent flexibility of such plants that permits a high level of response to uncertain market conditions and requirements. This level of response does require the use of efficient tools to help the decision-making process at the design and operational level. This paper presents a Mixed Integer Linear Program (MILP) model to optimise the scheduling of batch facilities subject to changeovers and distribution constraints so as to guarantee a pre-defined objective. Such an objective can be defined as the minimum orders' total lateness or the maximum distribution units loading capacity, among others. A continuous-time representation is used as well as the concept of job predecessor and successor to effectively handle changeovers. Facilities having non-identical parallel units/lines, sequence-dependent orders, finite release times for units and orders, restrictions on the suitability of jobs to lines/units and different possible destinations to available distribution units are also considered. Based on these characteristics the proposed model is able to determine the optimal allocation of jobs to production lines/units, the sequence of jobs on every line/unit and the starting and completion production times of each order. Also, the usage and allocation of the distribution resources (eg trucks) to orders and destinations are obtained based on their availability and suitability to the orders. The model led to the development of a prototype information system that can be used as a tool to help the decision-making process at the operational plant level.

Finally, the applicability of the proposed system/formulation is shown through the resolution of an industrial real case where the production of polymers is performed.

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1. Introduction

The short-term scheduling of batch facilities appears as an important and often complex industrial problem. The activity of such plants can be described as the manufacture of a wide range of products that must satisfy given amounts and due dates.

Operational restrictions such as pre-ordering, changeovers and maintenance constraints are often present and a non-regular production pattern is followed. Furthermore, within real plants, distribution restrictions (eg availability of trucks) influence the final orders due dates and should not be ignored when scheduling production.

In the last two decades, a large number of papers has been published in the area of planning and scheduling of batch facilities. Extensive reviews can be found in Reklaitis,[1] Applequist *et al*[2] and Shah.[3] The scheduling approaches reported can be grouped into two classes according to the treatment given to the time-domain representation. These can be respectively a discrete or a continuous time representation. On the former Kondili *et al*[4] proposed a quite general model based on the State-Task Network, STN framework. This allows the modelling of generic industrial problems where non-regular production patterns exist. This framework was later on used by several authors where more detailed models were developed accounting for many scheduling characteristics such as different operational and storage modes, changeovers, operational restrictions and batch mixing and splitting.[5,6] This type of formulation, however, presents some drawbacks which are related on the one hand to the time-domain approximation and on the other to the large-scale problems obtained when addressing real industrial cases.

In order to overcome some of these disadvantages continuous-time formulations have been reported, Pinto and Grossmann,[7] Cerda *et al*,[8] Schilling and Pantelides[9] and Castro *et al*[10] among others.

The above works have studied the scheduling problem with more or less detail; however, none has considered the case where distribution restrictions exist. These type of restrictions often influence the final orders' due dates and therefore should not be ignored when planning and scheduling production. This is evident for the cases where the final product inventory should be kept to a minimum.

In this paper, a continuous time model is developed to address this problem. This is based on the work of Cerda *et al.*[8] A mathematical formulation is proposed where two operational stages are considered. The first stage corre-

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sponds to the transformation of the raw materials into products — processing stage — and the second stage models the storage/dispatch of these same products.

Facilities having non-identical parallel units/lines, sequence-dependent orders, finite release times for units and orders, restrictions on the suitability of jobs to lines/units and maintenance requirements are considered. Moreover, the usage and allocation of the available distribution resources are studied.

The proposed model is then able to determine the optimal allocation of jobs to production lines/units, the sequence of jobs on every line/unit and the starting and completion production times of each order. The utilisation and allocation of the distribution resources (eg trucks) is also obtained based on its availability and suitability to orders.

Using the model described a prototype system was developed and it will be described later on in this paper.

Finally, the applicability of the proposed system/formulation is illustrated through the resolution of an industrial real case involving the production of polymers.

Problem characteristics

The short-term scheduling problem for multi-product batch facilities as studied in this paper, involves two operational stages. These are a processing stage with several in-phase parallel units and a product storage/dispatch stage where a warehouse and different distribution units are considered. The following features of each one of the stages are assumed:

Processing stage

- Each customer order or job involves a single product.
- Each order can be manufactured in a subset of available equipment items.
- A due date and size for each customer order has been specified. The manufacture of a product to inventory, without a particular due date, is also considered by assigning to it the last day of the planning horizon as a fictitious due date.
- The production is organised by product campaigns where batches of the same order are successively processed in the same unit.
- A non-pre-emptive mode of operation is assumed. Thus, a new order can only start in a certain unit after the previous order being processed in the same unit has been completed.
- The processing time for order j at unit/line $p(Tp_{jp})$ depends on both the nature of the order j and the type of unit/line p.
- Before starting a new campaign, a changeover period for cleaning or other type of equipment set-up can be present.
- The changeover time for a pair of orders i and $j(TS_{ij})$ may depend not only on the nature of the order but also on the order sequence (i, j).
- Preventive maintenance periods may be present during the time horizon in each line/unit.
- A maintenance period depends on the equipment maintainability and can be scheduled before, during and after unit/line changeover period.
- Forbidden jobs sequences at any equipment item due to operational restrictions (eg incompatible colours) are considered.
- A finite release time can be defined for units/lines as well as for orders.
- Due to limited storage capacity, intermediate receivers of raw materials may exist.

Product storage/dispatch stage

- Limited capacity is assumed for the product warehouse unit as well as for the distribution units.
- A particular distribution unit can be assigned to a delivery destination only if the total size of orders allocated to the distribution unit/delivery destination pair is within a minimum and maximum allowed capacity.
- Each distribution unit has a unique destination but can deliver customer orders to a different destination if the latter is close enough and the amount transported is within a specified material interval bound.
- The final order release times are defined by the release time of the truck to which the order has been allocated.

Based on these characteristics the problem goal is to find the optimal plant scheduling and the optimal storage/dispatch plan that guarantees a minimum total order delay or maximum loading distribution while satisfying the plant and distribution resources restrictions.

A MILP mathematical formulation was derived where a continuous time-representation was assumed. Equipment changeovers, maintenance requirements and operational as well as distribution constraints were considered.

The model is characterised by different types of binary and continuous variables, which are then combined so as to define the problem constraints.

Problem variables

To generate a production schedule, several operational decisions are to be made concerning the assignment of orders to units and the sequencing of orders and maintenance activities in each equipment item. Also, in order to create a product storage/delivery plan the allocation of orders to trucks and the trucks destination assignment has to be determined. The following types of binary variables represent such decisions:

- $X_{ijp}=1$ if the processing of order i takes place in the unit p just before order j; 0 otherwise.
- $Xp_{jp}=1$ if the processing of order j is the first being processed in unit p; 0 otherwise.

- $Xa_{j,a} = 1$ if order j is assigned to storage/distribution unit a; 0 otherwise.
- $Xd_{bd} = 1$ if distribution unit b is assigned to destination d; 0 otherwise.

Additionally, several non-negative continuous variables are also introduced. These define the campaign and the storage/dispatch timetable as well as the distribution resources usage along the time horizon considered.

- Ti_j = starting time of order j in the processing stage.
- $Tf1_j$ = completion time of order j in the processing stage.
- $Tf2_j$ = total completion time of order j — delivery time.
- $D2_j$ = delay in the delivery of order j.
- Qtd_{bd} = total loading of the distribution unit b assigned to product destination d.
- Qte_{bdc} = loading of distribution unit b going to destination d, which can deliver to destination c — truck nearby destination.

Problem constraints

Using the above variables the problem characteristics are modelled through the following sets of constraints:

- *Processing units orders assignment constraints*: define the assignment of orders to processing lines/units considering that:
  - (a) every order j has at most a unique predecessor i being manufactured just before in the same unit p;
  - (b) every order j presents at most a unique successor k in the job sequence of the assigned unit p;
  - (c) exactly one single order must be first processed in each equipment item;
  - (d) both the predecessor and the successor of a given order j must be manufactured in the same processing unit.
- *Processing stage time constraints*: define orders starting and completion time within the processing stage.
- *Maintenance constraints*: define the time occurrence maintenance activities and the associated conditions.
- *Distribution units orders assignment*: model the assignment of orders to distribution/storage units considering that:
  - (a) every order j can at most be assigned to a unique distribution/storage unit a;
  - (b) every distribution unit b can at most be assigned to a unique destination d;
  - (c) each distribution unit b allocated to delivery destination d can carry orders with a different destination c if within a pre-defined amount.
- *Distribution units loading*: relate the orders amount assigned to a certain distribution unit with the latter minimum and maximum capacity requirements.
- *Destination allocation constraints*: state that every distribution unit b has at most one unique destination d.
- *Dispatch stage time constraints*: define the completion times of orders during the storage/dispatch stage.
- *Processing/distribution constraints*: relates the two operational stages in terms of orders times availability.
- *Lateness constraints*: define the lateness in the completion of order j.

The objective function

As referred above, two alternative scheduling criteria have been defined. These are respectively: (1) the minimum total lateness and (2) the maximum loading capacity of the distribution units.

All constraints are expressed in linear form as well as the objective function, resulting in a MILP problem. This is solved by standard Branch and Bound package (XPRESS) as will be discussed later on.

System/interface description

In order to facilitate the use of the model developed a prototype information system was developed — *Scheduling Soft*. The main menu of this system is shown in Figure 1 where the user can select two basic options: optimise and parameter. These correspond, respectively, to the choice of the type of objective to be used and to the introduction of the problem data.

Thus, if the parameter option is chosen a new window appears (General Parameters Window — Figure 2) from where all the required parameters can be introduced into the system. These data are divided into order parameters (Figure 3), unit/line parameters (Figure 4), distribution parameters (Figure 5) and destination parameters (Figure 6).

In the general parameter window (Figure 2), the user is required to define the number of total orders, orders to be processed, activities maintenance, processing units, distribution units and destinations predicted for the horizon period under study. After this step, the system application creates the necessary sets for the problem. These sets are later filled

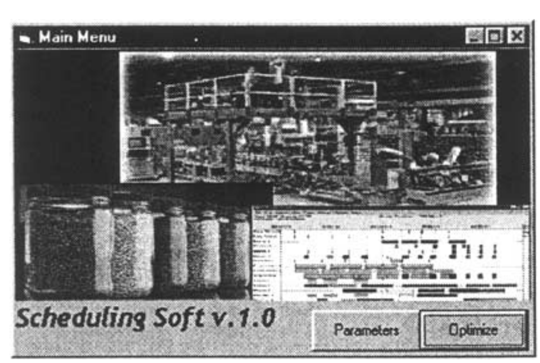

Figure 1 Scheduling Soft Main Menu.

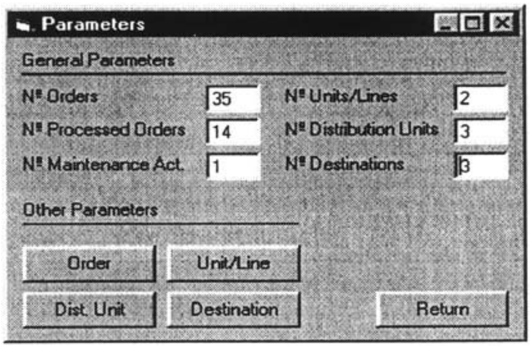

Figure 2 General Parameters Window.

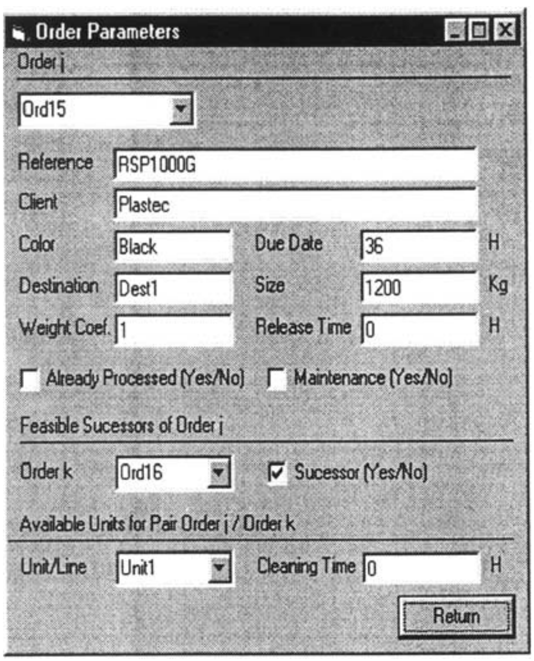

Figure 3 Order Parameters Window.

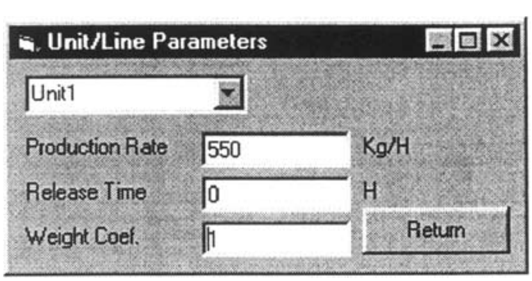

Figure 4 Line/Unit Parameters Window.

using the data corresponding to the orders, unit/lines, distribution units and destinations as specified, respectively, in Figures 3, 4, 5 and 6

After defining the problem data the user must choose the type of objective to be used (see Figure 7). This choice

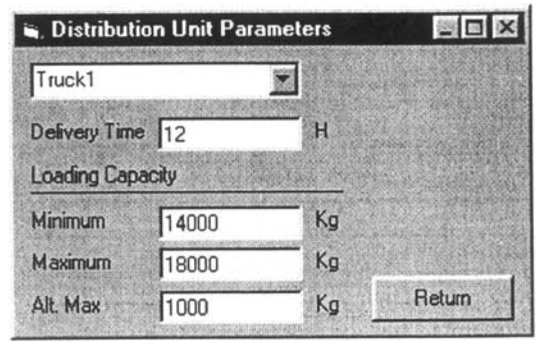

Figure 5 Distribution Unit Parameters Window.

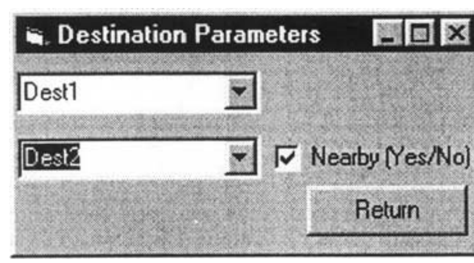

Figure 6 Destination Parameters Window.

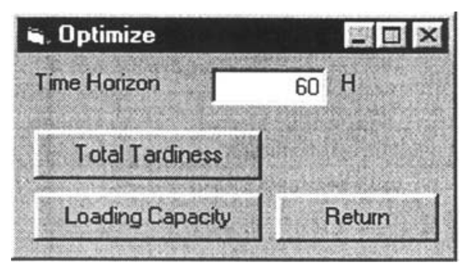

Figure 7 Optimise Window.

automatically activates the re-solution of the problem in study.

As final results the user can access the optimal scheduling plan (Figure 8) and the storage and dispatch plan (Figure 9).

Case study

The applicability of the proposed model/system is shown through an industrial case study of a polymer industry. The production system is formed by two stages, respectively, a processing and dispatch stage. The former involves two parallel extruders (Unit1 and Unit2) with unequal processing rates while the latter is characterised by a limited capacity warehouse and three distribution units (Truck1, Truck2 and Truck3).

A total of 34 orders (Ord1…Ord34) have to be delivered in order to satisfy customers' due dates. At the beginning of the time horizon ($T = 60$ h) studied, 14 orders have already

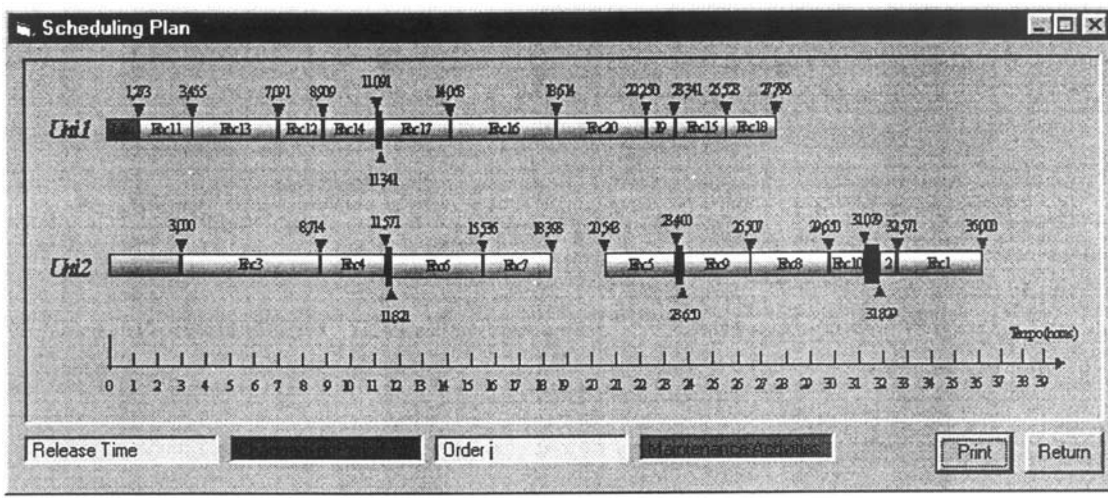

Figure 8 Scheduling Plan Window.

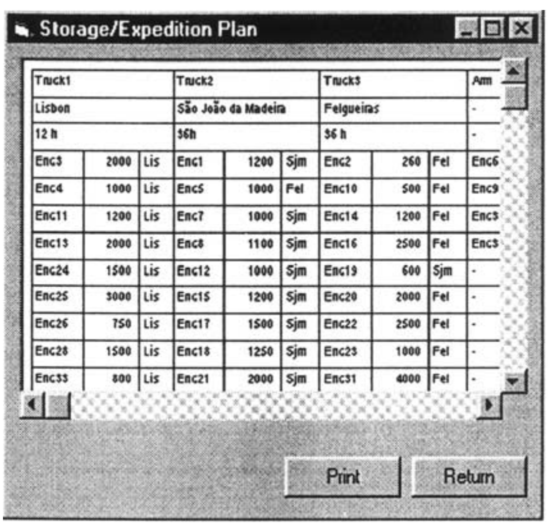

Figure 9 Storage/Dispatch plan.

been processed and are in storage. Thus, only 20 orders have to be scheduled within the plant processing stage. Furthermore, one maintenance activity has to be scheduled for Unit1. Within the second operational stage the allocation of the distribution units is to be made considering three delivery destinations: Felgueiras (Fel), Lisbon (Lis) and São João da Madeira (Sjm). The destination, size and due date orders are illustrated in Table 1.

Based on the plant procedures it is assumed that Unit1 processes only dark colours (black and brown colour orders) while Unit2 produces all the colours with the exception of the black colour. Orders Ord1 – Ord4 are colourless, Ord5 – Ord7 are white, Ord8 – Ord10 are yellow, Ord11 – Ord14 are brown and Ord15 – Ord20 are black. Therefore, orders Ord1 – Ord14 can be processed in Unit2 and Ord11 – Ord20 can be done in Unit1.

The processing rates of Unit1 and Unit2 are, respectively, of 550 and 350 kg/h. Unit2 is only available in the third hour of production — release time = 3 h — while Ord2 due to raw

Table 1 Due dates, delivery destinations and size of orders

Ord j	*Dest.*	De_j *(h)*	Qe_j *(kg)*	*Ord j*	*Dest.*	De_j *(h)*	Qe_j *(kg)*	*Ord j*	*Dest.*	De_j *(h)*	Qe_j *(kg)*
Ord1	Sjm	12	1200	Ord13	Lis	12	2000	Ord25	Lis	36	3000
Ord2	Fel	36	260	Ord14	Fel	60	1200	Ord26	Lis	36	750
Ord3	Lis	36	2000	Ord15	Sjm	36	1200	Ord27	Sjm	36	2800
Ord4	Lis	36	1000	Ord16	Fel	36	2500	Ord28	Lis	36	1500
Ord5	Fel	36	1000	Ord17	Sjm	36	1500	Ord29	Sjm	36	1500
Ord6	Fel	36	1300	Ord18	Sjm	36	1250	Ord30	Sjm	36	2500
Ord7	Sjm	36	1000	Ord19	Sjm	36	600	Ord31	Fel	36	4000
Ord8	Sjm	36	1100	Ord20	Fel	36	2000	Ord32	Fel	36	700
Ord9	Fel	36	1000	Ord21	Sjm	36	2000	Ord33	Lis	36	800
Ord10	Fel	36	500	Ord22	Fel	36	2500	Ord34	Sjm	36	350
Ord11	Lis	36	1200	Ord23	Fel	36	1000	Man	—	36	700
Ord12	Sjm	36	1000	Ord24	Lis	36	1500	—	—	—	—

Table 2 Computational results — portable PC with a AMD-K7 processor

Case	*1*	*2*
Constraints	1345	3157
Continuous variables	500	784
Binary variables	170	476
Best solution	48 h	45 210 kg
Tol. (%)	0	1.1
CPU times	282 s	181 s

material unavailability can only be processed after 10 h of production — release time = 10 h.

The set-up time for each pair of orders is based on colour switch and on the processing type equipment, that is, for Unit1 the brown/black colour change will require 0.1 h of set-up while the opposite changeover takes 0.25 h. In Unit2, every switch between colourless orders and another colour order takes 0.25 h of set-up. The remaining set-up times are: white to yellow or brown 0.25 h; white to colourless 0.5 h, yellow to colourless or white 0.75 h, yellow to brown 0.25 h and brown to other colour 0.75 h.

The minimum and maximum allowed loading capacity for each truck are 14 000 and 16 000 kg, respectively, while the maximum alternative loading capacity is 1000 kg.

The problem was solved for both objective functions, generating two cases. Case 1 for the minimum total delay and case 2 for the maximum loading capacity of trucks.

The computational results are shown in Table 2. Both problems were solved with the information system described above where the XPRESS optimiser was used to solve the MILP model. A portable computer with an AMD-K7 processor was used.

For case 1, a total of 282 CPUs was required to solve the problem to optimality while 181 CPUs were consumed in case 2. In the latter, the solution presented an integrality gap of 1.1%. The final schedule and storage/dispatch plans for the two objective functions are shown in Figures 10 and 11 and Tables 3 and 4.

The results reported appear as quite good, therefore, the proposed model seems promising for solving the scheduling of multi-product batch plants where not only operational constraints are considered but also distribution restrictions are incorporated.

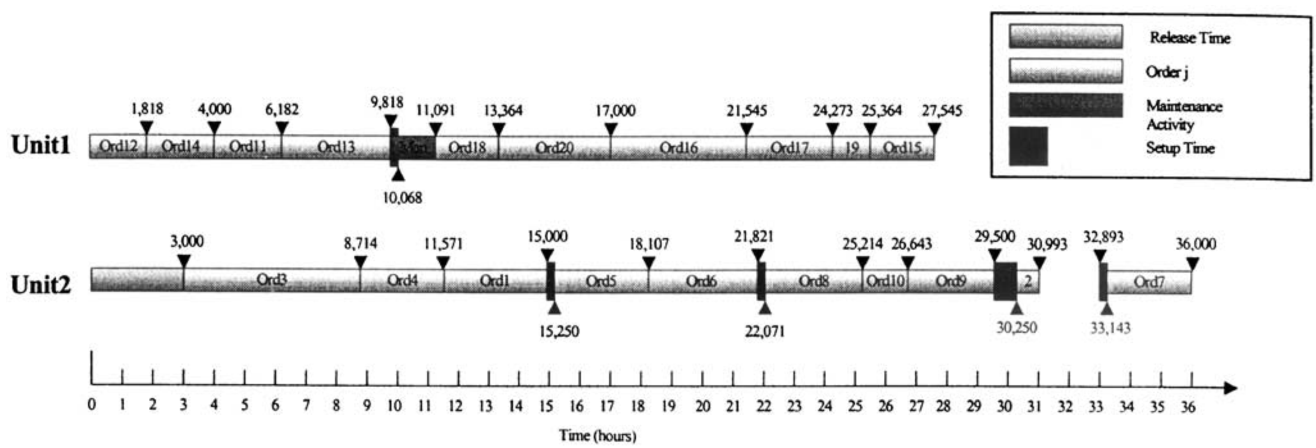

Figure 10 Scheduling plan for the total lateness objective.

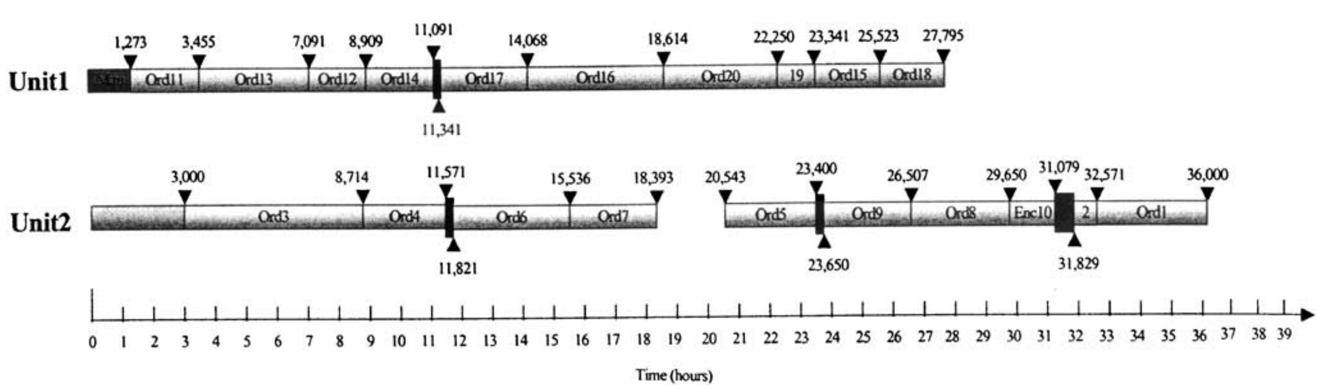

Figure 11 Scheduling plan for the total loading capacity objective.

Table 3 Storage/dispatch plan for total lateness objective

Distribution unit	Truck 1	Destination	Lisbon	$Tsa_a = 12$ h
Orders	Ord3, Ord4, Ord11, Ord13, Ord24, Ord25, Ord26, Ord28 and Ord33			
Distribution unit	Truck2	Destination	Felgueiras	$Tsa_a = 36$ h
Orders	Ord5, Ord6, Ord9, Ord16, Ord20, Ord22, Ord23, Ord31 and Ord32			
Distribution unit	Truck3	Destination	São João da Madeira	$Tsa_a = 36$ h
Orders	Ord1, Ord2, Ord7, Ord8, Ord10, Ord12, Ord15, Ord17, Ord18, Ord19, Ord21, Ord29, Ord30 and Ord34			
Storage unit	Warehouse	Destination	—	$Tsa_a = 60$ h
Orders	Ord14 and Ord27			

Table 4 Storage/dispatch plan for total loading capacity objective

Distribution unit	Truck1	Destination	Lisbon	$Tsa_a = 12$ h
Orders	Ord3, Ord4, Ord11, Ord13, Ord24, Ord25, Ord26, Ord28 and Ord33			
Distribution unit	Truck2	Destination	São João da Madeira	$Tsa_a = 36$ h
Orders	Ord1, Ord5, Ord7, Ord8, Ord12, Ord15, Ord17, Ord18, Ord21, Ord27 and Ord29			
Distribution unit	Truck3	Destination	Felgueiras	$Tsa_a = 36$ h
Orders	Ord2, Ord10, Ord14, Ord16, Ord19, Ord20, Ord22, Ord23, Ord31 and Ord32			
Storage Unit	Warehouse	Destination	—	$Tsa_a = 60$ h
Orders	Ord6, Ord9, Ord30 and Ord34			

Conclusions

The scheduling of batch multi-product facilities subject to operational changeovers and distribution constraints was addressed in this paper through the development of a mathematical model. The proposed model is able to determine the optimal allocation of jobs to production lines/units, the sequence of jobs on every line/unit and the starting and completion production times of each order. Furthermore, the usage and allocation of the distribution resources (eg trucks) is obtained based on its availability and suitability to orders. The latter is characterised by a certain amount and destination.

In order to facilitate the model utilisation a prototype system was developed. This appears as a promising starting point to the development of an efficient tool to help the decision-making process at the operational level in multi-product batch plants.

Finally, the applicability of the proposed system/formulation was illustrated through the resolution of an industrial real case describing the production of polymers.

Good results were obtained and therefore it can be stated that the proposed model appears as quite efficient.

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